The beauty of mathematics

Johannes Kepler built upon the vision of Copernicus and the observations of Brahe to formulate the three basic rules of celestial mechanics. By Martin Barraclough.

The man who built on the work of Copernicus and Brahe, Johannes Kepler, was born at Wiel-der-Stadt near Würtemberg, Germany, on December 27, 1571. He was the eldest of seven children, three of whom died during childhood. Johannes himself had smallpox and when he was six, and he continued to be a sickly child and, very disadvantageously as we might suppose for an astronomer, he suffered from blurred and defective vision. Perhaps this is one reason why he was always attracted to the mathematical, rather than the purely observational aspects of astronomy.

His family was, as we now call it, 'downwardly mobile'. Johannes had to cope with a strange mother who was later tried for witchcraft, some bullying by his siblings and eventual desertion by his father.

As the Dukes of Württemberg had set up a system of grants and scholorships for poor and gifted children of the Protestant faith, Kepler was lucky, despite his background, to have access to a fine education. Eventually, Kepler attended Tübingen University from 1591 to 1594 pursuing a doctorate in theology, but before he finished, he was diverted to a teaching position in mathematics in Graz. While at Tübingen University, however, he had read about the heliocentric universe of Copernicus.

Johannes Kepler 1571-1630. Picture courtesy Dee Levers.

Whatever his early life had been, Kepler became noticed in academic circles for an inspired piece of work which subsequently turned out to be completely wrong. Then as now it is sometimes possible to be right for the wrong reasons, and in his book the *Mysterium Cosmographicum*, published in 1597, he explained a concept which first struck him in a flash of imaginative insight. In this book Kepler argued that one could fit the five regular solids: cube, tetrahedron, octahedron, dodecahedron and icosahedron neatly between the orbits of the then six known planets. It was in this text that Kepler also made reference to the Copernican heliocentric universe.

Though we now know that Kepler's insight is wrong, it had the effect of engaging one of the greatest astronomical minds of all time with the theoretical problem of how the planets move in space.

Only a few years earlier, Tycho Brahe himself had discredited the ancient belief that the planets were carried around the heavens on transparent spheres, and if these spheres were no more, the problem remained: 'how did the planets move?' Kepler believed that they did so because of the action of an invisible force, which he mistakenly associated with magnetism, and did so in accordance with exact geometrical laws.

The *Mysterium Cosmographicum* also brought Kepler to the attention of Tycho Brahe whose observational data would provide the mathematical material from which Kepler would eventually extract his three laws.

Kepler's sufferings continued through his early manhood. He was encumbered by poverty (the school at Graz was eventually closed down by the Archduke Ferdinand of Hapsburg), an unhappy marriage, the death of some of his children and long-standing ill health. He was also in danger of religious persecution, being a devout Lutheran Protestant living in Roman Catholic Austria and Czechoslovakia – the Holy Roman Empire, as it was then called.

In 1599 it must have been with some relief when he was taken in by Tycho

"I do not speak like I write, I do not write like I think, I do not think like I ought to think." Johannes Kepler (1571-1630). It is fortunate that this complicated genius thought at all – his work finally set the Earth spinning free.

Brahe as an assistant and he and his family ntered Tycho's castle at Prague. The two ears Kepler spent with Tycho differ in interpretation depending on author, and whether Brahe and Kepler ever got on is a matter of debate. Both men had strong beliefs and personalities, and while Kepler wished to pursue his own mathematical problems, Tycho wanted Kepler to assist him in his research.

What was interesting to Kepler was ycho's rich treasury of observations of the planets that went back over nearly thirty years. When Tycho died suddenly in 1601, Kepler was able to retain that section of Tycho's observations upon which he was then working: the reduction of the complex orbit of Mars.

On the death of Tycho Brahe, Kepler came the Imperial Mathematician to mperor Rudolph II with a large salary that was never paid. He now spent much of his time casting horoscopes for the Emperor Rudolph and as he actually got paid for his horoscopes, astrology became important to his economic survival. But when not calculating horoscopes he struggled with the rbit of Mars.

After six years of trial and error he ealised that the only orbital shape that would fit Tycho's excellent observational data was the now familiar ellipse. He developed this new concept of elliptical, as opposed to circular planetary rotation and published the first two of his famous three laws of motion in 1609.

These two laws hastened the end of the d astronomical system and reinforced the principle that if two theories fit the data, the simpler one is the more beautiful and more likely to be accepted.

As important as these laws were, however, Kepler's work which made the most dramatic impact was yet to come. This was his third law published in his *Harmonices Mundi* (or *Harmony of the World*) in 1619 and which explained the mathematical symmetry of the Universe.

On the death of Rudolph II in 1612, Kepler moved to Linz and continued to work as a mathematician. He spent his last two years in Sagan.

Kepler died of a fever in Regensberg on November 15, 1630 after surviving yet more poverty, wars, a siege, and a further marriage. He was responsible for three major achievements which included the overturning of centuries of widely held beliefs in circular planetary orbits and provided the framework and inspiration for one of the greatest thinkers of any age, Isaac Newton.

It is perhaps a remarkable irony that had Kepler's life been an easier one, his science may have been the poorer and as a result, so would we.

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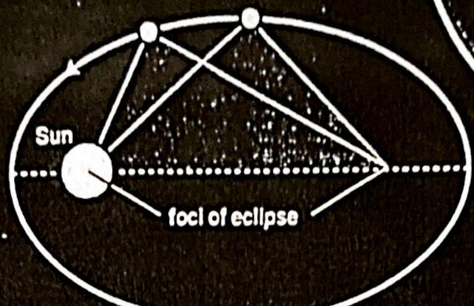
Kepler's laws of planetary motion

Kepler's first law

Each planet travels round the Sun in an elliptical orbit with the Sun at one focus of the ellipse. (below)

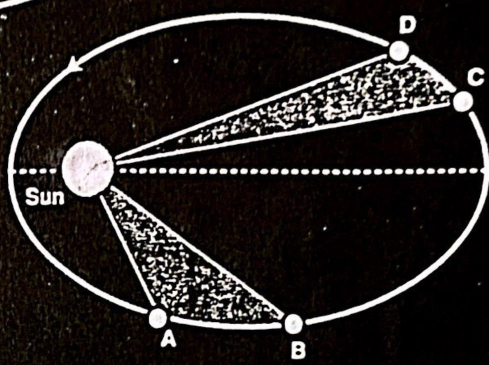
The point of closest approach to the Sun is called perihelion and the point at which the planet is furthest away is aphelion.

Johannes Kepler



Kepler's second law

The radius vector sweeps out equal areas of space in equal times (right).



Kepler's third law

The square of the orbital period is directly proportional to the cube of the planet's mean distance from the Sun.

'Mean distance' in this context denotes the semi-major axis of the orbital ellipse. If distance is measured in astronomical units (the astronomical unit, AU, is the semi-major axis of the Earth's orbit = 149,600,000km) and time in years, the relationship between period (P) and mean distance (a) can be written simply as:

$$P^2 = a^3$$

Demonstration of Kepler's third law:

Planet	Sidereal period (years)	Semi-major axis (AU)	P^2	$\bar{a}^3$
Mercury	0.24	0.39	0.06	0.06
Venus	0.61	0.72	0.37	0.37
Earth	1.00	1.00	1.00	1.00
Mars	1.88	1.52	3.53	3.51
Jupiter	11.86	5.20	140.7	140.6
Saturn	29.46	9.54	867.9	868.3

Kepler's laws are obeyed in any situation where two objects orbit each other under the influence of their mutual gravitational attraction. This can be planets orbiting a parent star, two stars orbiting each other or artificial satellites orbiting the Earth.